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Crosstalk in Coaxial Cables—Analysis Based on Short-Circuited and Open Tertiaries

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The problem considered herein is that of estimating, from measurements on short lengths of coaxial cable, the crosstalk to be expected in long lengths of the same cable. The method developed, which is particularly applicable to cases in which the effect of tertiary circuits on the crosstalk is large, is based on measurements of crosstalk in a short length, with the tertiaries first short-circuited and then open. The application of this method to the cable described in the companion paper by Messrs. Booth and Odarenko gave crosstalk values in good agreement with their experimental results.

Introduction

FOR a number of years the problem of crosstalk summation in long open-wire lines or cables has been studied by measuring crosstalk, in phase and magnitude, in short lengths. The crosstalk within a short length, between two circuits terminated in their characteristic impedances, would be measured with all important tertiary circuits also approximately terminated. Then the crosstalk between two circuits in adjoining short lengths would be measured with the tertiary circuits terminated. From these coefficient measurements the crosstalk in a long length could be estimated by a process of integration.

The application of this general method to crosstalk in the usual types of coaxial cable would require great accuracy in the coefficient measurements, because in longer lengths the desired crosstalk value depends on the difference between two nearly equal quantities involving the coefficients. In the following analysis the computation of the crosstalk for long lengths of coaxial cable is based on crosstalk measurements, in phase and magnitude, between two coaxial circuits in a single short length with the tertiary circuits first open and then short-circuited, no crosstalk measurements with terminated tertiary circuits being involved.

This method of analysis, when applied to the twin coaxial cable described in the companion paper by Messrs. Booth and Odarenko, gave results in good agreement with the measured crosstalk.

In this analysis it was assumed that all the tertiary circuits could be combined and considered as a single circuit. Although no evidence has been found that with the types of structure studied so far, better accuracy would result from the further refinement of considering two or more dissimilar tertiary circuits with coupling between them, there is one case of practical importance which cannot be handled with the single-tertiary analysis. This case is that of the interaction crosstalk (that is, the crosstalk by way of a tertiary circuit) between two adjoining lengths of coaxial cable, when, at the junction, part of the tertiary conductors are short-circuited to the outer coaxial conductors while the remaining tertiary conductors continue through with no discontinuity. This problem might be of importance where, at a repeater, the outer coaxial conductors and the sheath are bonded together, but paper-insulated pairs in the same sheath provide an uninterrupted tertiary circuit. The near-end crosstalk under such conditions might also differ significantly from the values indicated by the single-tertiary analysis.

The two-tertiary analysis is too long to be given here in detail, and hence has been outlined only to such an extent as to indicate the derivation of the formulas for interaction crosstalk when one of the tertiaries is short circuited and the other terminated in its characteristic impedance. The formula for near-end crosstalk under this condition is given without derivation.

I—Identical Coaxial Lines Symmetrically Placed with Respect to a Single Tertiary

The first case we shall consider herein is that of any number of identical coaxial lines with the outer coaxial conductors in continuous electrical contact and symmetrically placed with respect to a single tertiary circuit, such as that which might be provided by a sheath surrounding the coaxial lines and insulated from the outer coaxial conductors, or by a surrounding layer of paper-insulated pairs. Throughout we shall assume that the reaction of the induced currents upon the disturbing line is negligible.

Following a nomenclature analogous to that of the Schelkunoff-Odarenko paper,[1] we will designate by Z_{12} the mutual impedance per unit length between any two coaxial lines in the presence of the other coaxial lines but in the absence of any other conductors. The mutual

[1] *Bell System Technical Journal*, April, 1937.

impedance per unit length between any coaxial line (in the presence of the other coaxial lines) and the tertiary circuit consisting of all of the coaxial outer conductors with return by way of the sheath or other tertiary conductors, we will designate by Z_{13}.

If we consider the crosstalk between two coaxial lines of length, l, such that the coaxial lines and the tertiary are electrically short, each coaxial line being terminated in its characteristic impedance Z and the tertiary open at each end, the crosstalk (near-end and far-end being identical for such a length) is given by

$$\frac{Z_{12}l}{2Z}.$$

If, now, we consider a case similar except that the tertiary is short-circuited at each end, the crosstalk is the above term plus the effect of the tertiary current I_3, which, for unit current in the disturbing coaxial line, is given by

$$I_3 = \frac{Z_{13}}{Z_{33}},$$

where Z_{33} is the series impedance of the tertiary circuit per unit length. This tertiary current will produce a current $\left(-\frac{Z^2{}_{13}l}{2ZZ_{33}}\right)$ in the disturbed coaxial line and the total crosstalk will be

$$\frac{Z_{12}l}{2Z} - \frac{Z^2{}_{13}l}{2ZZ_{33}}.$$

If we designate $\frac{Z_{12}}{2Z}$ by X and $\frac{Z^2{}_{13}}{Z_{12}Z_{33}}$ by ξ, then, for an electrically short length, X will represent the crosstalk per unit length between two coaxial lines with the tertiary open, and $X(1-\xi)$ the crosstalk per unit length with the tertiary short-circuited. In the formulas developed below these quantities will be found to be of fundamental importance.

Tertiary Terminated in its Characteristic Impedance

Far-End Crosstalk

From the Schelkunoff-Odarenko paper, the sum of the direct far-end crosstalk (eq. 19) and the indirect far-end crosstalk (eq. 40) for any length under these conditions gives the total far-end crosstalk F_t, as

$$F_t = \frac{Z_{12}l}{2Z} - \frac{Z^2{}_{13}}{4ZZ_3}\left[\frac{2\gamma_3 l}{\gamma_3^2-\gamma^2} - \frac{1-e^{-(\gamma_3-\gamma)l}}{(\gamma_3-\gamma)^2} - \frac{1-e^{-(\gamma_3+\gamma)l}}{(\gamma_3+\gamma)^2}\right], \quad (1)$$

where

Z_3 = characteristic impedance of tertiary circuit,
γ, γ_3 = propagation constants of coaxial lines and tertiary circuit respectively.

This may be rearranged and written (since $Z_3\gamma_3 = Z_{33}$)

$$F_t = \frac{Z_{12}l}{2Z} - \frac{Z^2_{13}l}{2ZZ_{33}} - \frac{Z^2_{13}}{2ZZ_{33}} \times \left[\frac{l\gamma^2}{\gamma_3^2 - \gamma^2} - \frac{\gamma_3}{2}\left(\frac{1 - e^{-(\gamma_3-\gamma)l}}{(\gamma_3 - \gamma)^2} + \frac{1 - e^{-(\gamma_3+\gamma)l}}{(\gamma_3 + \gamma)^2} \right) \right] \tag{2a}$$

$$= X\left[l(1 - \xi) - l\xi \frac{\gamma^2}{\gamma_3^2 - \gamma^2} + \frac{\xi\gamma_3}{2} \times \left(\frac{1 - e^{-(\gamma_3-\gamma)l}}{(\gamma_3 - \gamma)^2} + \frac{1 - e^{-(\gamma_3+\gamma)l}}{(\gamma_3 + \gamma)^2} \right) \right]. \tag{2b}$$

This formula has been found to be applicable, with good accuracy, to the types of coaxial cable which have been studied so far. The quantities X and $X(1 - \xi)$ are determined from crosstalk measurements on a short length, and the propagation constants are of course readily determined.

Near-End Crosstalk

A similar approach to the problem of the near-end crosstalk N_t with the tertiary terminated in its characteristic impedance, using equations (10) and (32) of the Schelkunoff-Odarenko paper, gives

$$N_t = X\left[(1 - e^{-2\gamma l})\left(\frac{1 - \xi}{2\gamma} - \frac{\xi\gamma}{2(\gamma_3^2 - \gamma^2)} \right) + \frac{\xi\gamma_3}{2(\gamma_3^2 - \gamma^2)}(1 + e^{-2\gamma l} - 2e^{-(\gamma_3+\gamma)l}) \right]. \tag{3}$$

Here, as in the case of equation (2b) above, the crosstalk may be computed readily from crosstalk and impedance measurements on a short sample.

Interaction Crosstalk

Far-End Far-End and Far-End Near-End

We will consider the interaction crosstalk between two adjoining sections of lengths l and l', respectively, the tertiary being connected through at the junction, with no discontinuity. The tertiary current $i_3(l)$ at the far end of a section of length l, for unit sending-end current, with the tertiary terminated in its characteristic impedance, is readily formulated as

$$i_3(l) = \frac{Z_{13}}{2Z_3}\frac{e^{-\gamma l} - e^{-\gamma_3 l}}{\gamma_3 - \gamma}. \tag{4}$$

In the adjoining section, with the tertiary terminated in its characteristic impedance, the tertiary current $i_3(y)$ will be given by $i_3(l)e^{-\gamma_3 y}$, where y is the distance measured from the junction of the two sections.

This tertiary current $i_3(y)$ will produce a far-end current in the disturbed coaxial of

$$\frac{Z^2_{13}}{4ZZ_3}\frac{e^{-\gamma l} - e^{-\gamma_3 l}}{\gamma_3 - \gamma}\int_0^{l'} e^{-\gamma_3 y}e^{-\gamma(l'-y)}dy.$$

The equal-level far end-far end interaction crosstalk FF, being this far-end current divided by $e^{-\gamma(l+l')}$, may be obtained as

$$FF = \frac{X\xi\gamma_3}{2(\gamma_3 - \gamma)^2}(1 - e^{-(\gamma_3-\gamma)l})(1 - e^{-(\gamma_3-\gamma)l'}). \tag{5}$$

The near-end current in the disturbed coaxial due to the current $i_3(y)$ is given by

$$\frac{Z^2_{13}}{4ZZ_3}\frac{e^{-\gamma l} - e^{-\gamma_3 l}}{\gamma_3 - \gamma}\int_0^{l'} e^{-\gamma_3 y}e^{-\gamma y}dy.$$

From this the equal-level far end-near end interaction crosstalk FN, being this near-end current divided by $e^{-\gamma l}$, may be obtained as

$$FN = \frac{X\xi\gamma_3}{2(\gamma_3^2 - \gamma^2)}(1 - e^{-(\gamma_3-\gamma)l})(1 - e^{-(\gamma_3+\gamma)l'}). \tag{6}$$

Near-End Near-End

The near-end tertiary current in the section of length l is similarly formulated as

$$i_3(0) = \frac{Z_{13}}{2Z_3}\frac{1 - e^{-(\gamma_3+\gamma)l}}{\gamma_3 + \gamma}. \tag{7}$$

The near-end near-end interaction crosstalk NN is readily obtained, in a fashion similar to that outlined above for the far-end near-end interaction crosstalk, as

$$NN = \frac{X\xi\gamma_3}{2(\gamma_3 + \gamma)^2}(1 - e^{-(\gamma_3+\gamma)l})(1 - e^{-(\gamma_3+\gamma)l'}). \tag{8}$$

Tertiary Short-Circuited

The general case of the crosstalk between coaxial lines of length l with the tertiary short-circuited at each end may be attacked as follows. At any point at a distance x from the sending end, the voltage gradient along the outer surface of the outer coaxial conductors, for unit sending-end current, will be $Z_{13}e^{-\gamma x}$. Each differential element, $Z_{13}e^{-\gamma x}dx$, of this voltage drop will produce a current in the tertiary circuit de-

termined by the impedances, Z' and Z'', of the tertiary as seen in the two directions from this point, these impedances being

$$Z' = Z_3 \tanh \gamma_3 x \tag{9}$$

and

$$Z'' = Z_3 \tanh \gamma_3(l - x) \tag{10}$$

respectively toward and away from the sending end.

At any other point at a distance y from the sending end, the tertiary current due to the voltage $Z_{13}e^{-\gamma x}dx$ will be given, for $y > x$, by

$$i_3(y) = \frac{Z_{13}e^{-\gamma x}dx}{Z' + Z''}\frac{\cosh \gamma_3(l - y)}{\cosh \gamma_3(l - x)}. \tag{11}$$

From this the transfer admittance $A(x, y)$ between these two points is obtained as

$$A(x, y) = \frac{1}{Z_3}\frac{\cosh \gamma_3(l - y)}{\tanh \gamma_3 x \cosh \gamma_3(l - x) + \sinh \gamma_3(l - x)}. \tag{12}$$

Similarly, for $y < x$, this transfer admittance is obtained as

$$A(x, y) = \frac{1}{Z_3}\frac{\cosh \gamma_3 y}{\sinh \gamma_3 x + \cosh \gamma_3 x \tanh \gamma_3(l - x)}. \tag{13}$$

The tertiary current, $i_3(x)$, is given by

$$i_3(x) = \int_0^l Z_{13}e^{-\gamma y}A(x, y)dy \tag{14}$$

from which we obtain

$$i_3(x) = \frac{Z_{13}}{2Z_3 \sinh \gamma_3 l} \times \left[\begin{array}{l} \dfrac{1}{\gamma_3 + \gamma}\left[\begin{array}{c}\cosh \gamma_3(l - x) + e^{-\gamma x} \sinh \gamma_3 l \\ - e^{-\gamma l} \cosh \gamma_3 x\end{array}\right] \\ - \dfrac{1}{\gamma_3 - \gamma}\left[\begin{array}{c}\cosh \gamma_3(l - x) - e^{-\gamma x} \sinh \gamma_3 l \\ - e^{-\gamma l} \cosh \gamma_3 x\end{array}\right]\end{array}\right]. \tag{15}$$

Far-End Crosstalk

The indirect far-end crosstalk F_s' due to this tertiary current (eq. 15) is given by

$$F_s' = e^{\gamma l}\int_0^l \frac{Z_{13}i_3(x)}{2Z} e^{-\gamma(l-x)}dx \tag{16a}$$

$$= \frac{Z^2_{13}}{2ZZ_{33}}\left[\frac{l\gamma_3^2}{\gamma_3^2 - \gamma^2} - \frac{2\gamma_3\gamma^2}{(\gamma_3^2 - \gamma^2)^2}\frac{\cosh \gamma_3 l - \cosh \gamma l}{\sinh \gamma_3 l}\right]. \tag{16b}$$

If this is combined with the direct far-end crosstalk $\frac{Z_{12}l}{2Z}$, and the terms rearranged as in the case of equation (2b), the total far-end crosstalk F_s is obtained as

$$F_s = X\left[l(1-\xi) - l\xi\frac{\gamma^2}{\gamma_3^2-\gamma^2} + 2\xi\frac{\gamma_3\gamma^2}{(\gamma_3^2-\gamma^2)^2}\frac{\cosh\gamma_3 l - \cosh\gamma l}{\sinh\gamma_3 l}\right]. \quad (17)$$

It will be noted that equations (2b) and (17) differ only in the terms which are not proportional to the length and which thus are of decreasing importance as the length becomes great.

Near-End Crosstalk

The indirect near-end crosstalk N_s' due to the tertiary current $i_3(x)$ is given by

$$N_s' = \int_0^l \frac{Z_{13}i_3(x)}{2Z}e^{-\gamma x}dx \quad (18)$$

By substituting $i_3(x)$ from equation (15) herein, and combining the result with the direct near-end crosstalk,

$$\frac{Z_{12}}{2Z}\frac{1-e^{-2\gamma l}}{2\gamma}$$

we obtain the total near-end crosstalk N_s, which may be written in the form

$$N_s = X\left[\begin{array}{l}\dfrac{1-e^{-2\gamma l}}{2\gamma}\left((1-\xi)+\dfrac{\xi\gamma^2(\gamma_3^2+\gamma^2)}{(\gamma_3^2-\gamma^2)^2}\right)\\ -\left(\dfrac{\xi\gamma_3\gamma^2}{(\gamma_3^2-\gamma^2)^2}\right)\left(\dfrac{(1+e^{-2\gamma l})\cosh\gamma_3 l - 2e^{-\gamma l}}{\sinh\gamma_3 l}\right)\end{array}\right]. \quad (19)$$

II—Identical Coaxial Lines Symmetrically Placed with Respect to Each of Two Dissimilar Tertiaries

We will now consider the case of any number of identical coaxial lines with the outer conductors in continuous electrical contact and symmetrically placed with respect to each of two dissimilar tertiaries with coupling between them.

In an unpublished memorandum by J. Riordan, the general forms are developed for the currents and voltages in two parallel circuits having uniformly distributed self and mutual impedances and admittances, when these circuits are subjected to impressed axial fields.

These currents (I_1 and I_2) and voltages (V_1 and V_2) (the subscripts applying, of course, to the respective tertiaries) are given by the coefficient array,

	$(a_1 + P_1)e^{-\gamma_1 x}$	$(b_1 + Q_1)e^{\gamma_1 x}$	$(a_2 + P_2)e^{-\gamma_2 x}$	$(b_2 + Q_2)e^{\gamma_2 x}$
I_1	1	-1	η_2	$-\eta_2$
I_2	η_1	$-\eta_1$	1	-1
V_1	K_1	K_1	$-\eta_1 K_2$	$-\eta_1 K_2$
V_2	$-\eta_2 K_1$	$-\eta_2 K_1$	K_2	K_2

where a_1, b_1, a_2 and b_2 are constants to be determined from the boundary conditions, and

$$\frac{P_1}{Q_1} = \frac{1}{2K_1(1 - \eta_1\eta_2)} \int e^{\pm\gamma_1 x}(f_1 + \eta_1 f_2)dx, \tag{20}$$

$$\frac{P_2}{Q_2} = \frac{1}{2K_2(1 - \eta_1\eta_2)} \int e^{\pm\gamma_2 x}(\eta_2 f_1 + f_2)dx, \tag{21}$$

f_1 and f_2 being the impressed fields along circuits 1 and 2 respectively.

If we consider the two tertiary circuits as consisting of (1) the outer coaxial conductors in parallel with return by tertiary path 1 and (2) tertiary path 1 — tertiary path 2, only tertiary circuit 1 will be subjected to an impressed field. Thus we will have $f_2 = 0$ and $f_1 = Z_{13}e^{-\gamma x}$ (for unit sending-end current in the disturbing coaxial line), where Z_{13} is the mutual impedance, per unit length, between a coaxial (in the presence of the other paralleling coaxials) and tertiary 1, and γ is the propagation constant, per unit length, for the coaxial circuit. The other quantities in this array are circuit parameters given as follows in terms of the series impedances Z_{11}, Z_{22} and Z_{12} per unit length and admittances a_{11}, a_{22} and a_{12} per unit length (subscripts 11 and 22 for self impedance or self admittance of circuits 1 and 2 respectively, and 12 for mutuals):

$$\begin{aligned}\frac{\gamma_1^2}{\gamma_2^2} = \tfrac{1}{2}[a_{11}Z_{11} + a_{22}Z_{22} + 2a_{12}Z_{12} \pm ((a_{11}Z_{11} - a_{22}Z_{22})^2 \\ + 4(a_{11}Z_{12} + a_{12}Z_{22})(a_{12}Z_{11} + a_{22}Z_{12}))^{1/2}],\end{aligned} \tag{22}$$

$$\eta_1 = \frac{\gamma_1^2 - a_{11}Z_{11} - a_{12}Z_{12}}{a_{11}Z_{12} + a_{12}Z_{22}}, \tag{23}$$

$$\eta_2 = \frac{\gamma_2^2 - a_{12}Z_{12} - a_{22}Z_{22}}{a_{12}Z_{11} + a_{22}Z_{12}}, \tag{24}$$

$$K_1 = \frac{Z_{11} + \eta_1 Z_{12}}{\gamma_1} = \frac{\gamma_1}{a_{11} - \eta_2 a_{12}}, \tag{25}$$

$$K_2 = \frac{Z_{22} + \eta_2 Z_{12}}{\gamma_2} = \frac{\gamma_2}{a_{22} - \eta_1 a_{12}}. \tag{26}$$

From equations (20) and (21) above, we have

$$P_1 = \frac{Z_{13}}{2K_1(1 - \eta_1\eta_2)(\gamma_1 - \gamma)} e^{(\gamma_1 - \gamma)x}, \tag{27}$$

$$Q_1 = \frac{-Z_{13}}{2K_1(1 - \eta_1\eta_2)(\gamma_1 + \gamma)} e^{-(\gamma_1 + \gamma)x}, \tag{28}$$

$$P_2 = \frac{Z_{13}\eta_2}{2K_2(1 - \eta_1\eta_2)(\gamma_2 - \gamma)} e^{(\gamma_2 - \gamma)x}, \tag{29}$$

$$Q_2 = \frac{-Z_{13}\eta_2}{2K_2(1 - \eta_1\eta_2)(\gamma_2 + \gamma)} e^{-(\gamma_2 + \gamma)x}. \tag{30}$$

If we designate

$$\frac{Z_{13}}{K_1(1 - \eta_1\eta_2)(\gamma_1^2 - \gamma^2)} \text{ by } \psi_1$$

and

$$\frac{Z_{13}\eta_2}{K_2(1 - \eta_1\eta_2)(\gamma_2^2 - \gamma^2)} \text{ by } \psi_2,$$

we have

$$I_1 = a_1 e^{-\gamma_1 x} - b_1 e^{\gamma_1 x} + \eta_2 a_2 e^{-\gamma_2 x} - \eta_2 b_2 e^{\gamma_2 x} + (\psi_1\gamma_1 + \psi_2\eta_2\gamma_2)e^{-\gamma x}, \tag{31}$$

$$I_2 = \eta_1 a_1 e^{-\gamma_1 x} - \eta_1 b_1 e^{\gamma_1 x} + a_2 e^{-\gamma_2 x} - b_2 e^{\gamma_2 x} + (\psi_1\eta_1\gamma_1 + \psi_2\gamma_2)e^{-\gamma x}, \tag{32}$$

$$V_1 = K_1 a_1 e^{-\gamma_1 x} + K_1 b_1 e^{\gamma_1 x} - \eta_1 K_2 a_2 e^{-\gamma_2 x} - \eta_1 K_2 b_2 e^{\gamma_2 x} + (\psi_1 K_1 \gamma - \psi_2 K_2 \eta_1 \gamma)e^{-\gamma x}, \tag{33}$$

$$V_2 = -\eta_2 K_1 a_1 e^{-\gamma_1 x} - \eta_2 K_1 b_1 e^{\gamma_1 x} + K_2 a_2 e^{-\gamma_2 x} + K_2 b_2 e^{\gamma_2 x} - (\psi_1 K_1 \eta_2 \gamma - \psi_2 K_2 \gamma)e^{-\gamma x}. \tag{34}$$

Before proceeding with the application of these results to specific crosstalk problems, we will establish certain relations which, as in the single-tertiary analysis, will be fundamental in relating crosstalk measurements on short lengths of cable to the crosstalk to be expected in a longer length.

Let us consider the crosstalk as measured on a short length under the following two conditions: (1) both tertiaries open and (2) tertiary 1 short-circuited at each end and tertiary 2 open. We will designate the crosstalk under condition (1) by Xl and under condition (2) by $Xl(1 - \xi)$. Under condition (2) the tertiary current (I_1) for unit current in the energized coaxial is given by $\frac{Z_{13}}{Z_{11}}$ and the indirect crosstalk current in the disturbed coaxial is thus $\frac{-Z^2_{13}l}{2ZZ_{11}}$, so that we have

$$X\xi = \frac{Z^2_{13}}{2ZZ_{11}}. \tag{35}$$

Interaction Crosstalk with One Tertiary Short-Circuited

Far-End

For the sake of simplicity, and with no considerable loss of applicability, we will postulate the restriction that $e^{\gamma_1 l}$ and $e^{\gamma_2 l}$ are large compared with $e^{\gamma l}$, where l is the length of the section in which we are formulating the tertiary currents.

Referring to equations (31) to (34), under the above restrictions the terms involving $e^{-\gamma_1 x}$ and $e^{-\gamma_2 x}$ are negligible in the region near $x = l$ and thus in this region

$$I_1 = -b_1 e^{\gamma_1 x} - \eta_2 b_2 e^{\gamma_2 x} + [(p_1 - q_1) + \eta_2(p_2 - q_2)]e^{-\gamma x}, \tag{36}$$

$$I_2 = -\eta_1 b_1 e^{\gamma_1 x} - b_2 e^{\gamma_2 x} + [\eta_1(p_1 - q_1) + (p_2 - q_2)]e^{-\gamma x}, \tag{37}$$

$$V_1 = K_1 b_1 e^{\gamma_1 x} - \eta_1 K_2 b_2 e^{\gamma_2 x} + [K_1(p_1 + q_1) - \eta_1 K_2(p_2 + q_2)]e^{-\gamma x}, \tag{38}$$

$$V_2 = -\eta_2 K_1 b_1 e^{\gamma_1 x} + K_2 b_2 e^{\gamma_2 x} + [-\eta_2 K_1(p_1 + q_1) + K_2(p_2 + q_2)]e^{-\gamma x}, \tag{39}$$

where [2]

$$p_1 = \frac{Z_{13}}{2K_1(1 - \eta_1\eta_2)(\gamma_1 - \gamma)}, \tag{40}$$

$$q_1 = \frac{-Z_{13}}{2K_1(1 - \eta_1\eta_2)(\gamma_1 + \gamma)}, \tag{41}$$

$$p_2 = \frac{Z_{13}\eta_2}{2K_2(1 - \eta_1\eta_2)(\gamma_2 - \gamma)}, \tag{42}$$

$$q_2 = \frac{-Z_{13}\eta_2}{2K_2(1 - \eta_1\eta_2)(\gamma_2 + \gamma)}. \tag{43}$$

If, now, x is measured from the far end, and the following substitutions are made as a matter of convenience: [3]

$$a_1 = b_1 e^{(\gamma_1 + \gamma)l}, \tag{44}$$

$$a_2 = b_2 e^{(\gamma_2 + \gamma)l}, \tag{45}$$

equations (36) to (39), multiplied by $e^{\gamma l}$ so that the currents and voltages are given for unit received current in the energized coaxial, become

$$I_1 = -a_1 e^{-\gamma_1 x} - \eta_2 a_2 e^{-\gamma_2 x} + [(p_1 - q_1) + \eta_2(p_2 - q_2)]e^{\gamma x}, \tag{46}$$

$$I_2 = -\eta_1 a_1 e^{-\gamma_1 x} - a_2 e^{-\gamma_2 x} + [\eta_1(p_1 - q_1) + (p_2 - q_2)]e^{\gamma x}, \tag{47}$$

[2] The terms involving $e^{-\gamma x}$ here are identical with the corresponding terms in equations (31) to (34) except for the change in nomenclature, which in each case has been chosen so that the a's and b's will be given by simple functions of the parameters employed (p's and q's here: ψ's in the previous equations).

[3] a_1 and a_2 here have no relation to a_1 and a_2 in equations (31) to (34).

$$V_1 = K_1 a_1 e^{-\gamma_1 x} - \eta_1 K_2 a_2 e^{-\gamma_2 x} + [K_1(p_1 + q_1) - \eta_1 K_2(p_2 + q_2)]e^{\gamma x}, \quad (48)$$

$$V_2 = -\eta_2 K_1 a_1 e^{-\gamma_1 x} + K_2 a_2 e^{-\gamma_2 x} + [-\eta_2 K_1(p_1 + q_1) + K_2(p_2 + q_2)]e^{\gamma x}. \quad (49)$$

In the section, of length l', adjacent to the far end of the energized section, the impressed fields are zero and thus (under the condition that $e^{\gamma_1 l'}$ and $e^{\gamma_2 l'}$ are large compared with unity), using primes to indicate currents and voltages in this region, with the distance x' taken positive from $x = l$,

$$I_1' = a_1' e^{-\gamma_1 x'} + \eta_2 a_2' e^{-\gamma_2 x'}, \quad (50)$$

$$I_2' = \eta_1 a_1' e^{-\gamma_1 x'} + a_2' e^{-\gamma_2 x'}, \quad (51)$$

$$V_1' = K_1 a_1' e^{-\gamma_1 x'} - \eta_1 K_2 a_2' e^{-\gamma_2 x'}, \quad (52)$$

$$V_2' = -\eta_2 K_1 a_1' e^{-\gamma_1 x'} + K_2 a_2' e^{-\gamma_2 x'}. \quad (53)$$

With tertiary 1 short-circuited, the boundary conditions to be satisfied are that at $x = x' = 0$, $V_1 = V_1' = 0$ and $I_2 = I_2'$. From these boundary conditions, we obtain

$$a_1 = -(p_1 + q_1) + \frac{\eta_1 K_2(\eta_1 p_1 + p_2)}{K_1 + \eta_1^2 K_2}, \quad (54)$$

$$a_2 = -(p_2 + q_2) + \frac{K_1(\eta_1 p_1 + p_2)}{K_1 + \eta_1^2 K_2}, \quad (55)$$

$$a_1' = \frac{\eta_1 K_2(\eta_1 p_1 + p_2)}{K_1 + \eta_1^2 K_2}, \quad (56)$$

$$a_2' = \frac{K_1(\eta_1 p_1 + p_2)}{K_1 + \eta_1^2 K_2}. \quad (57)$$

The equal-level far-end far-end interaction crosstalk FF_s is given by [4]

$$FF_s = \frac{Z_{13}}{2Z} e^{\gamma l'} \int_0^{l'} I_1' e^{-\gamma(l'-x')} dx'. \quad (58)$$

With I_1' as given by equations (50), (56) and (57), under the restrictions we have placed on $\gamma_1 l'$ and $\gamma_2 l'$, we have

$$FF_s = \frac{Z^2_{13}}{4Z(1 - \eta_1\eta_2)(K_1 + \eta_1^2 K_2)} \times \left[\frac{\eta_1}{K_1(\gamma_1 - \gamma)} + \frac{\eta_2}{K_2(\gamma_2 - \gamma)}\right]\left[\frac{\eta_1 K_2}{\gamma_1 - \gamma} + \frac{\eta_2 K_1}{\gamma_2 - \gamma}\right] \quad (59)$$

[4] As pointed out in the Schelkunoff-Odarenko paper in the section on mutual impedance, since a coaxial circuit is involved, the current distribution external to this circuit does not affect the mutual impedance, and hence the current I_2' contributes nothing to the crosstalk.

or, with the use of equation (35),

$$FF_s = \frac{X\xi Z_{11}}{2(1 - \eta_1\eta_2)(K_1 + \eta_1^2 K_2)} \times \left[\frac{\eta_1}{K_1(\gamma_1 - \gamma)} + \frac{\eta_2}{K_2(\gamma_2 - \gamma)}\right]\left[\frac{\eta_1 K_2}{\gamma_1 - \gamma} + \frac{\eta_2 K_1}{\gamma_2 - \gamma}\right]. \quad (60)$$

The equal-level far-end near-end interaction crosstalk FN_s is given by

$$FN_s = \frac{Z_{13}}{2Z}\int_0^{l'} I_1' e^{-\gamma x'} dx', \quad (61)$$

and under the restrictions we have placed on $\gamma_1 l'$ and $\gamma_2 l'$, we have

$$FN_s = \frac{Z^2_{13}}{4Z(1 - \eta_1\eta_2)(K_1 + \eta_1^2 K_2)} \times \left[\frac{\eta_1}{K_1(\gamma_1 - \gamma)} + \frac{\eta_2}{K_2(\gamma_2 - \gamma)}\right]\left[\frac{\eta_1 K_2}{\gamma_1 + \gamma} + \frac{\eta_2 K_1}{\gamma_2 + \gamma}\right] \quad (62a)$$

$$= \frac{X\xi Z_{11}}{2(1 - \eta_1\eta_2)(K_1 + \eta_1^2 K_2)} \times \left[\frac{\eta_1}{K_1(\gamma_1 - \gamma)} + \frac{\eta_2}{K_2(\gamma_2 - \gamma)}\right]\left[\frac{\eta_1 K_2}{\gamma_1 + \gamma} + \frac{\eta_2 K_1}{\gamma_2 + \gamma}\right]. \quad (62b)$$

Near-End

Under the above restriction that $e^{\gamma_1 l}$, $e^{\gamma_1 l'}$, $e^{\gamma_2 l}$ and $e^{\gamma_2 l'}$ are large compared with $e^{\gamma l}$, the currents and voltages in the disturbing section near $x = 0$ are given by equations (31) to (34) with the b-terms omitted, and the currents and voltages in the disturbed section adjacent to the sending end by equations (50) to (53).

The boundary conditions to be satisfied are that at $x = x' = 0$, $V_1 = V_1' = 0$ and $I_2 = -I_2'$. From these boundary conditions we obtain

$$a_1 = -(p_1 + q_1) + \frac{\eta_1 K_2(q_2 + \eta_1 q_1)}{K_1 + \eta_1^2 K_2}, \quad (63)$$

$$a_2 = -(p_2 + q_2) + \frac{K_1(q_2 + \eta_1 q_1)}{K_1 + \eta_1^2 K_2}, \quad (64)$$

$$a_1' = \frac{\eta_1 K_2(q_2 + \eta_1 q_1)}{K_1 + \eta_1^2 K_2}, \quad (65)$$

$$a_2' = \frac{K_1(q_2 + \eta_1 q_1)}{K_1 + \eta_1^2 K_2}. \quad (66)$$

The near-end near-end interaction crosstalk NN_s is given by

$$NN_s = \frac{Z_{13}}{2Z}\int_0^{l'} I_1' e^{-\gamma x} dx \tag{67a}$$

$$= -\frac{Z^2_{13}}{4Z(1-\eta_1\eta_2)(K_1+\eta_1^2K_2)} \times \left[\frac{\eta_1}{K_1(\gamma_1+\gamma)}+\frac{\eta_2}{K_2(\gamma_2+\gamma)}\right]\left[\frac{\eta_1K_2}{\gamma_1+\gamma}+\frac{\eta_2K_1}{\gamma_2+\gamma}\right] \tag{67b}$$

$$= -\frac{X\xi Z_{11}}{2(1-\eta_1\eta_2)(K_1+\eta_1^2K_2)} \times \left[\frac{\eta_1}{K_1(\gamma_1+\gamma)}+\frac{\eta_2}{K_2(\gamma_2+\gamma)}\right]\left[\frac{\eta_1K_2}{\gamma_1+\gamma}+\frac{\eta_2K_1}{\gamma_2+\gamma}\right]. \tag{67c}$$

Near-End Crosstalk with One Tertiary Short-Circuited

Although the derivation of the formula for near-end crosstalk N_s with one tertiary short-circuited is too long to be included here, it seems advisable to give this formula without derivation. Under the above mentioned restriction that $e^{\gamma_1 l}$ and $e^{\gamma_2 l}$ are large compared with $e^{\gamma l}$,

$$N_s = N_t + NN - NN_s, \tag{68}$$

where

N_t = near-end crosstalk, tertiaries terminated,

NN = near-end near-end interaction crosstalk between two adjoining lengths, tertiaries with no discontinuity,

NN_s = near-end near-end interaction crosstalk between two adjoining lengths with tertiary 1 short-circuited at the junction.

The first two terms (N_t and NN) may, with the types of cable studied so far, be determined with satisfactory accuracy from the single-tertiary analysis. In such a case, the formulas given herein are sufficient for computing the near-end crosstalk with one tertiary short-circuited.

III—Comparison of Computed Crosstalk with Measured Values

With 72-ft. and 145-ft. samples of the twin coaxial cable described in the companion paper by Messrs. Booth and Odarenko, crosstalk and impedance measurements were made in the laboratory, at frequencies from 50 kc to 300 kc, the sheath and quads in parallel being considered as providing a single tertiary, that is, as being connected together at short intervals.

The far-end crosstalk for a length of 5 miles was computed from these laboratory measurements and in Fig. 1 the results are compared with measurements on this length, the crosstalk in either case being practically the same whether the tertiary was terminated or short-circuited.

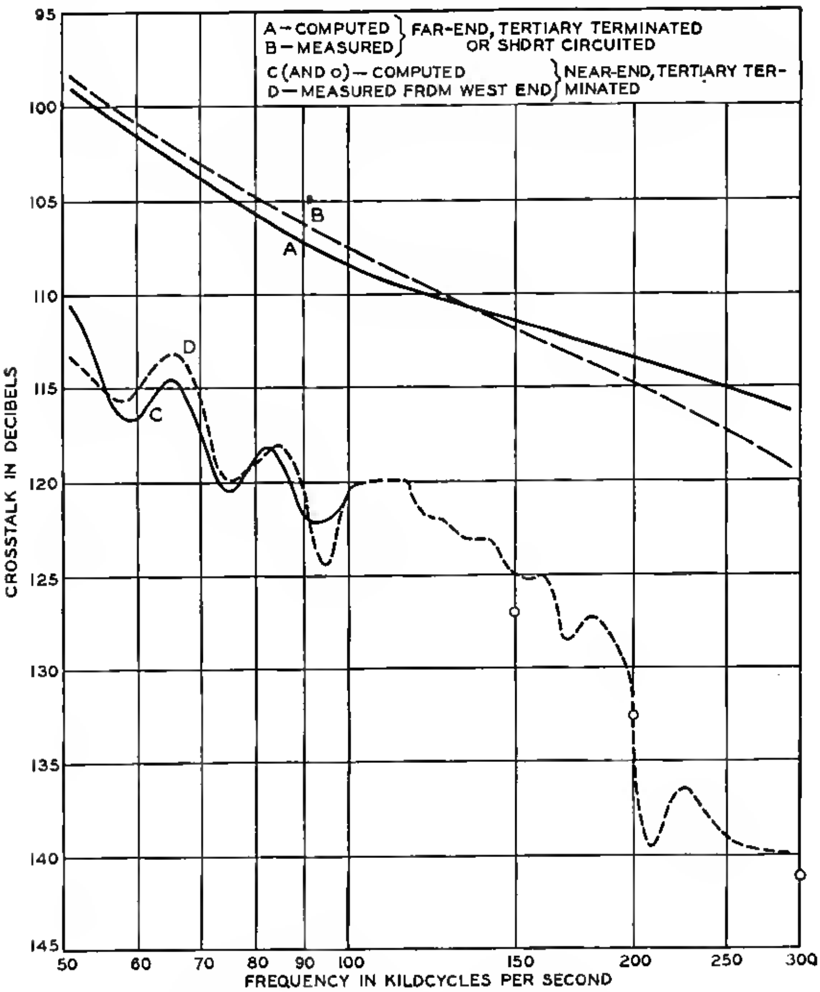

Fig. 1—Far-end crosstalk and near-end crosstalk for 5-mile length.

In Figs. 1 and 2, the computed near-end crosstalk for a length of 5 miles is compared with representative measurements on the above mentioned twin coaxial cable. Figure 1 shows this comparison with the tertiary terminated and Fig. 2 with the tertiary short-circuited.

The assumption of uniformity of the coaxial lines, as regards transfer impedances, and of the tertiary circuits as regards transmission characteristics, is a more serious restriction in the computation of near-end crosstalk than in the case of far-end crosstalk. Even for long lengths of cable, the near-end crosstalk is determined almost entirely by the crosstalk behavior of a relatively short length of the cable near the sending end, whereas the average crosstalk characteristics determine

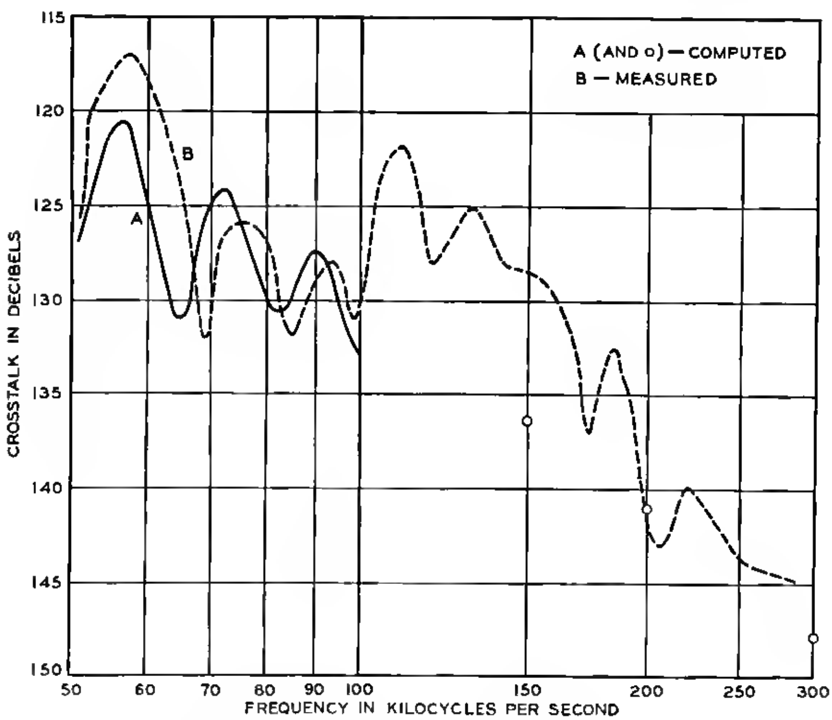

Fig. 2—Near-end crosstalk for 5-mile length, tertiary short-circuited.

the far-end crosstalk for a long length of cable. Thus, from measurements on representative short lengths, the far-end crosstalk for a long length may generally be computed more accurately than can the near-end crosstalk.

Similarly, the various types of interaction crosstalk depend largely upon the crosstalk behavior of relatively short lengths of the cable near the junction. In Fig. 3, the far-end far-end interaction crosstalk has been chosen as an illustration of the correlation which has been obtained between computed interaction crosstalk for the above men-

tioned twin coaxial cable and the measured interaction crosstalk The curves in Fig. 3 are for equal lengths either of 3000 ft. or 12,000 ft. In the case of the measured values, the junction point of the two sections was not the same for these two lengths. Although the agreement between calculated and measured values is only fair, the spread in the experimental results for these two cases, which for uniform cable would be slight, is about the same as the spread between calculated and measured values.

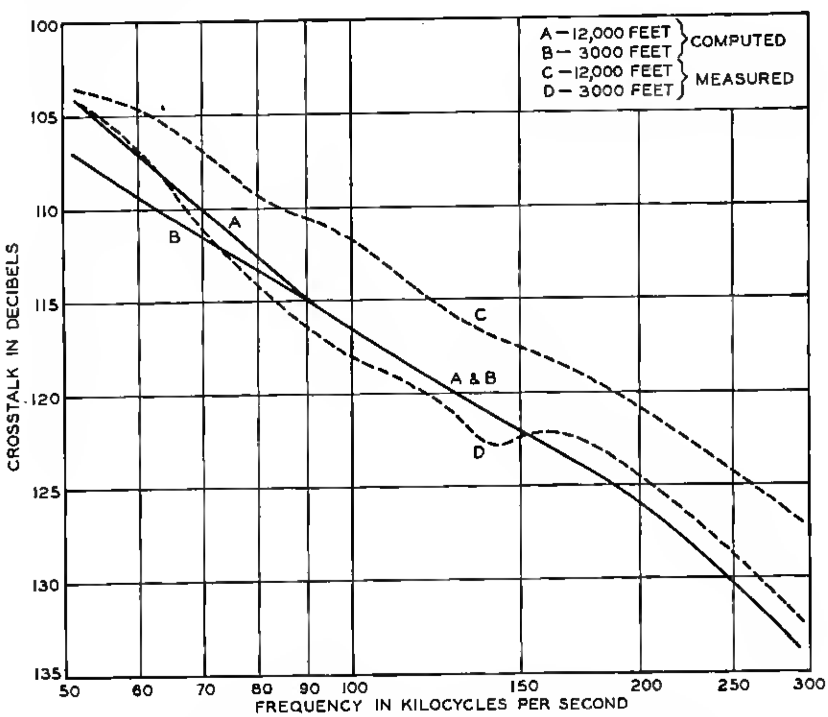

Fig. 3—Far-end far-end interaction crosstalk between two equal lengths.

The two-tertiary formulas have so far been applied only to one type of cable with four coaxial lines and a layer of paper-insulated pairs. The longest length of this type of cable on which crosstalk measurements have been made is 1900 ft. The various types of interaction crosstalk with one tertiary short-circuited, as computed from the formulas given above, agree roughly with the measured interaction crosstalk under this same condition. However, the restriction that the tertiary circuits involved are electrically long, as postulated in deriving the interaction crosstalk formulas for this case, is not satisfied,

and comparisons of the calculated and measured values are not very significant.

It may be remarked that the application of the single-tertiary analysis to all cases in which the two tertiaries were treated alike (either terminated or short-circuited) gave very satisfactory agreement between computed and measured crosstalk for this 1900 ft. length of 4-coaxial cable.